Chapter 4 - Day 3

Ex: let $g(x) = (x-2)^3$

a) find $g'(x)$

Note: $g(x) = (x-2)(x-2)(x-2) = x^3 - 6x^2 + 12x - 8$

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)^3 - 6(x+h)^2 + 12(x+h) - 8 - [x^3 - 6x^2 + 12x - 8]}{h}$$

$$= \lim_{h \to 0} \frac{\cancel{x^3} + 3hx^2 + 3h^2x + h^3 - \cancel{6x^2} - 12xh - 6h^2 + \cancel{12x} + 12h - \cancel{8} - \cancel{x^3} + \cancel{6x^2} - \cancel{12x} + \cancel{8}}{h}$$

$$= \lim_{h \to 0} \frac{3hx^2 + 3h^2x + h^3 - 12xh - 6h^2 + 12h}{h}$$

$$= \lim_{h \to 0} 3x^2 + 3hx + h^2 - 12x - 6h + 12$$

$$= 3x^2 + 3(0)x + 0^2 - 12x - 6(0) + 12$$

$= 3x^2 - 12x + 12$

$= 3(x^2 - 4x + 4)$

$= 3(x-2)^2$

b) find $g'(4)$ and $g'(-2)$

$g'(4) = 3(4-2)^2 = 3 \cdot 4 = 12$

$g'(-2) = 3(-2-2)^2 = 3 \cdot 16 = 48$

Ex: Suppose that

$$\frac{f(x+h)-f(x)}{h} = \frac{-2h(x+2)-h^2}{h(x+h+2)^2(x+2)^2}$$

Find the slope of the tangent line at $x=1$.

Goal: find $f'(1)$.

$$f'(1) = \lim_{h\to 0} \frac{f(1+h)-f(1)}{h}$$ *"plug 1 in for x"

$$f'(1) = \lim_{h\to 0} \frac{-2h(1+2)-h^2}{h(1+h+2)^2(1+2)^2}$$

$$= \lim_{h\to 0} \frac{-2h(3)-h^2}{h(h+3)^2(3)^2}$$

$$= \lim_{h\to 0} \frac{-6h-h^2}{9h(h+3)^2} = \lim_{h\to 0} \frac{-\cancel{h}(6+h)}{9\cancel{h}(h+3)^2}$$

$$= \lim_{h\to 0} \frac{-(6+h)}{9(h+3)^2} = \frac{-(6+0)}{9(0+3)^2} = \frac{-6}{81} = \boxed{\frac{-2}{27}}$$

Ex: Suppose $\frac{f(x+h)-f(x)}{h} = 5x+2h+3$

and $f(2)=1$. Find the equation of the tangent line to $f(x)$ at $x=2$.

* we need a point and a slope.

Point $(2,1)$

Slope $f'(2)$

$$f'(x) = \lim_{h\to 0} 5x+2h+3$$

$$= 5x+2(0)+3 = 5x+3$$

$$f'(2) = 5(2)+3 = 13$$

tangent line $y-1=13(x-2)$

$$y=13x-25$$

Ex: let $f(x) = \begin{cases} x^2 & x \le 3 \\ mx+b & x > 3 \end{cases}$

Find m and b such that f(x) is differentiable at x=3.

* This means we need f(x) differentiable and continuous at x=3.

"differentiable" = derivatives on both sides are equal.

if $f(x) = x^2$, $f'(x) = 2(1)x + 0 = 2x$

if $f(x) = mx+b$, $f'(x) = m$

for $x=3$, $2x = m$

$2(3) = m$

$\boxed{6 = m}$

"continuous" = function values on both sides are equal

so $x^2 = mx+b$

→

thus for $x=3$,

$$3^2 = m(3)+b$$

$$9 = 3m+b$$ but $m=6$

$$9 = 3(6)+b$$

$$9 = 18+b$$

$$\boxed{-9 = b}$$